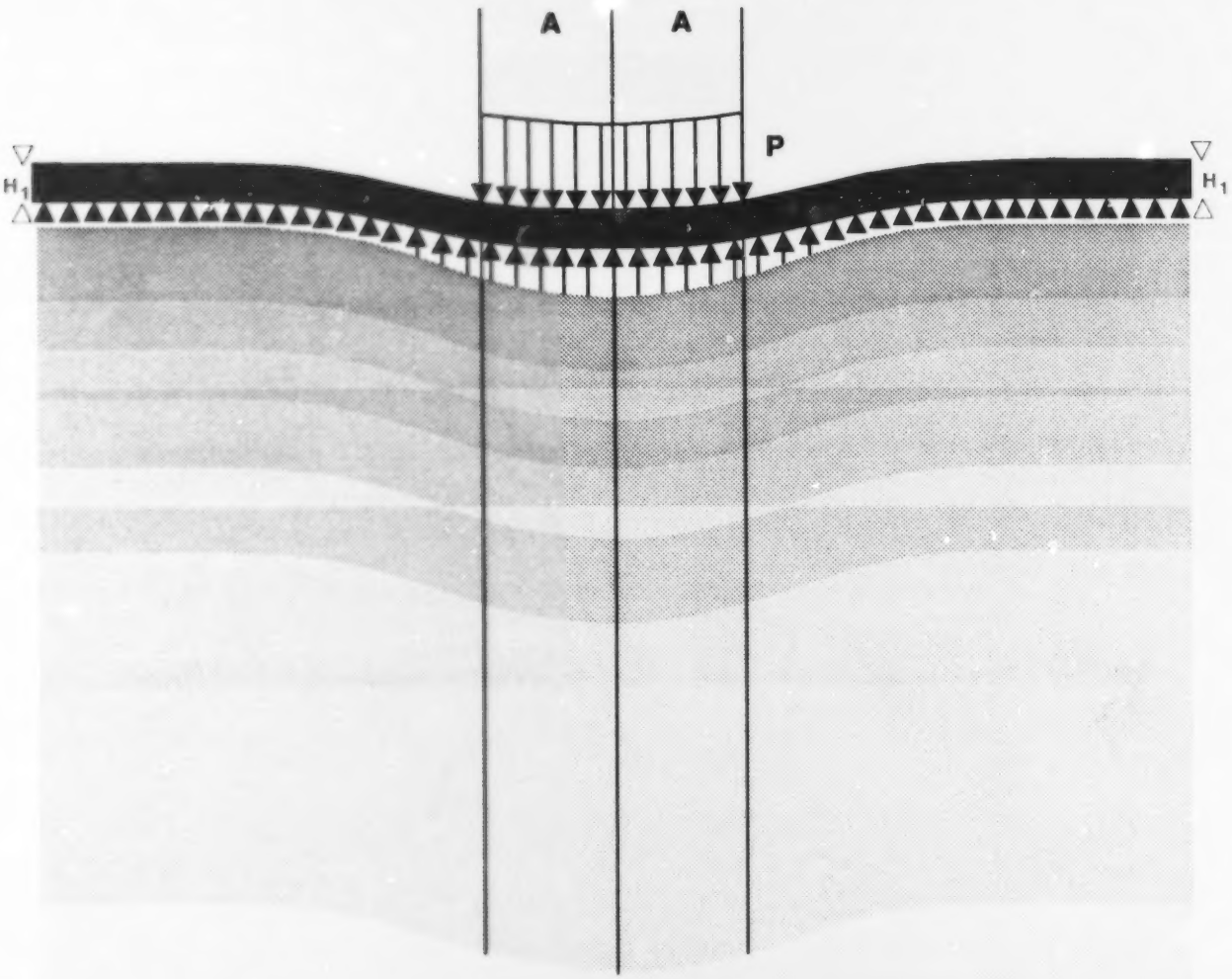
RR-242

Interpretation of Deflection Basin for Real-world Materials in Flexible Pavements



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Interpretation of Deflection Basin for Real-world Materials in Flexible Pavements

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Abstract:	The PROBE computer program was developed to calculate critical response parameters in the top layer and subgrade of flexible pavements, under the test load of a Falling Weight Deflectometer (FWD). The Boussinesq hyperbola, which describes surface deflections from a single concentrated static load at a distance from the load axis, was used to compute subgrade moduli. Thus, PROBE was originally modelled assuming a simple static model of elastic behavior. This original PROBE program was tested on an experimental site north of Toronto. Two adjacent pavement sections of different strengths were built on the same uniform subgrade. Their computed subgrade moduli were found to be different i.e., consistently larger at the test points of the stronger section. This constituted a systematic error, caused by the fact that the Boussinesq equation is not valid for many pavement materials. It is concluded that, for the interpretation of FWD measurements, backcalculation methods based on mere static models of elastic theories must be questioned. The problem was solved for the PROBE program by connecting Boussinesq's hyperbola to another hyperbola (with a non-linear term in the denominator) which can be numerically defined by curve-fitting the distant points of the deflection basin. The connecting point is the intersection of the two hyperbolas. This is the only point where a valid pair of distant/deflection values can be taken from the new hyperbola, for the Boussinesq-based calculation of subgrade moduli in the PROBE program. In this way the systematic error, i.e., the difference between the computed moduli of the adjacent test sections, was eliminated. Many consistency checks are presented to verify the quality of the program.
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1/ Introduction

Since 1987, a continuous effort has been made in Ontario to solve the problem of deflection basin interpretation of flexible pavements. In view of the complexity of pavement materials and the uncertainties of many design parameters, it was assumed that:

- a simple solution would serve as well as a complex one;
- a workable solution could be based on the assumption of linear elasticity of materials; and
- a static method of deflection calculation would sufficiently represent the propagation of a deflection wave from a falling weight deflectometer (FWD) impulse load.

In 1988, a first version of the computer program PROBE was completed and its concepts were presented at the First International Symposium on Nondestructive Testing of Pavements and Backcalculation, June 1988, sponsored by the American Society for Testing and Materials, and at the Annual Meeting of the Transportation Research Board, January 1989. During and after this period, this first PROBE program was applied to data from numerous FWD tests, especially to data from a well-controlled test section Highway 7N, north of Toronto. Many of the results were consistent with theoretical expectations; such as the very slight decrease of subgrade modulus with increases of test load, or the very slight change in all subgrade parameters in the face of large errors in estimating asphaltic concrete (AC) moduli (for the same load level and the same set of measured FWD deflections).

On the other hand, inconsistencies were discovered with regard to the calculated subgrade moduli. The subgrade moduli in adjacent pavement sections of different strengths should have been approximately the same at all test locations, because the sections were built on the same uniform subgrade. However, the computed moduli under the stronger section were consistently higher. The only explanation was that the deflections measured at a distance from the load axis were significantly affected by the elastoplastic or dynamic properties of real pavement materials. Thus, the effort had to continue.

The raw data used in this report was established by Trow Geotechnical Ltd., Brampton, Ontario.

2/ The Instrument

Measurement of an impulse deflection wave by the falling weight deflectometer appears to have emerged as the coming method of structural pavement evaluation and design.

The FWD instrument works as follows: A weight of known magnitude is dropped from different heights, creating various levels of impulse loads. The loads are transmitted to the pavement surface with the amount of damping controlled by springs and rubber pads. The test load is distributed over the surface by a rubber-padded circular platen or disk. The pavement structure responds by a dynamic wave of deflections which spreads outward from the centre under the load. The peaks of this deflection wave are measured at several surface points by *sensors* called geophones. One of the sensors is placed in the centre, accessible through a hole in the disk, and the others at various distances outside the disk. The outer sensors are placed on the pavement surface by lowering a boom. All sensors can be lifted for transportation after each test and put down again when measuring resumes.

The measured deflections generated by the FWD test load represent a deflection bowl or basin such as it may occur under a passing wheel load of corresponding magnitude and speed and of similar distribution area of tire contact pressure. The measurements are fast, efficient, and accurate (if the instrument is properly calibrated), and the data can be instantly recorded and processed.

3/ Depth and Shape of the Deflection Basin Curve

3.1/ Interpretation with Magnitude and Shape of the Deflection Bowl

For many years, deflections under a standard dual-tire wheel load, measured by the Benkelman Beam (BB) and adjusted for temperature, have been regarded as an indicator of the structural strength of flexible pavements. The Falling Weight Deflectometer measures not only one deflection under the test load, but provides information on the whole deflection basin under and around this load; it measures the peaks of an impulse load deflection wave. What essentially is new with FWD over BB is the additional information on the *shape* of the deflection basin.

Deflection bowls of the same depth can have a wide or a narrow shape. A wide shape would indicate a weak subgrade, whereas a narrow shape would point to weak upper layers in the pavement structure, refer to Figure 1, a) and b). The latter could be caused by high temperatures in the asphalt concrete layers which soften these upper layers and cause higher deflections at and near the centre, without affecting the more distant deflections, refer to Figure 1, c). A similar effect can be observed if the asphaltic layers are of low quality or cracked.

Thus, a qualitative interpretation can be made by evaluating the *basin shape* (in conjunction with the maximum deflection or basin depth). *Wide* means soft lower layers and/or a weak subgrade. *Narrow* means soft upper layers.

In quantitative terms, shape is interpreted with regard to the relative elastic stiffness of the different layers. The process of determining the elastic stiffness of layers from a given set of surface deflections (measured by FWD) is called *backcalculation* which involves the shape and magnitude of deflections.

3.2/ Specific Features about the Outer Part of the Deflection Basin

For a static single load and linear-elastic material, the deflections around the load follow a simple hyperbola with a linear term of distance in the denominator i.e., a Boussinesq hyperbola. This is true for surface deflections of the elastic half-space as well as for those of elastic layer systems such as deflections calculated with the ELSYM5 program, but in the latter case only for deflections *distant* from the load.

The distant deflections from FWD measurements on real-world layer materials often follow a different hyperbola, with a power term of distance in the denominator. The deflections decrease and die away faster than with the Boussinesq hyperbola. The difference between the two hyperbolas may indicate non-linearity of elasticity in real pavement materials. It may also be due to the difference between a static load and a dynamic impact load. Figure 2 illustrates the two models of material behaviour. Some pavements with cement- or lime-treated bases and those with old, brittle bituminous layers were observed to follow Boussinesq, whereas pavements with relatively new asphaltic materials follow the other hyperbola.

These specific features of deflections distant from the test load, are important for modelling a simple program for FWD data interpretation.

The grey curve in Figure 2 is the Boussinesq hyperbola of surface deflections for a linear-elastic half-space. The distant surface deflections calculated by programs of elastic layer analysis (ELSYM5) are adequately described by this hyperbola.

The solid curve in Figure 2 is a different hyperbola which describes the distant deflections measured by FWD as they line up on this new curve. Most pavements tested in Ontario were found to follow this curve with regard to their distant measured deflections. For more details refer to Section 6.2.

Figure 1/ Qualitative interpretation of deflection basin shape

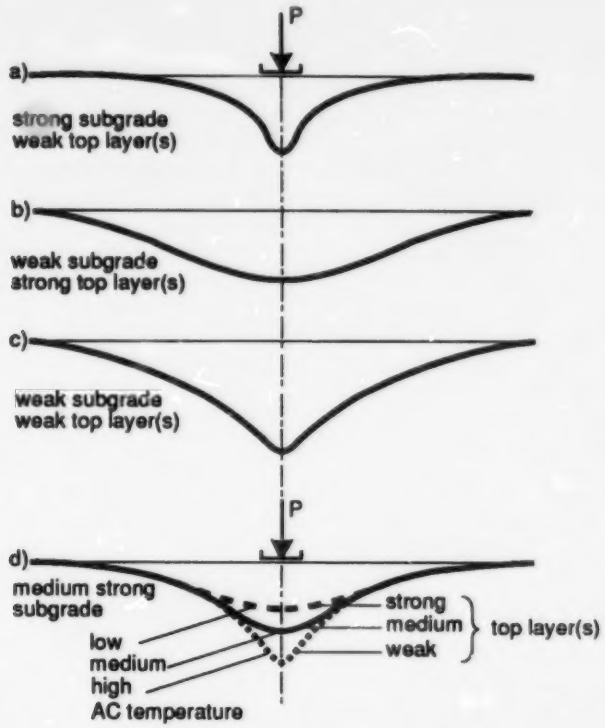
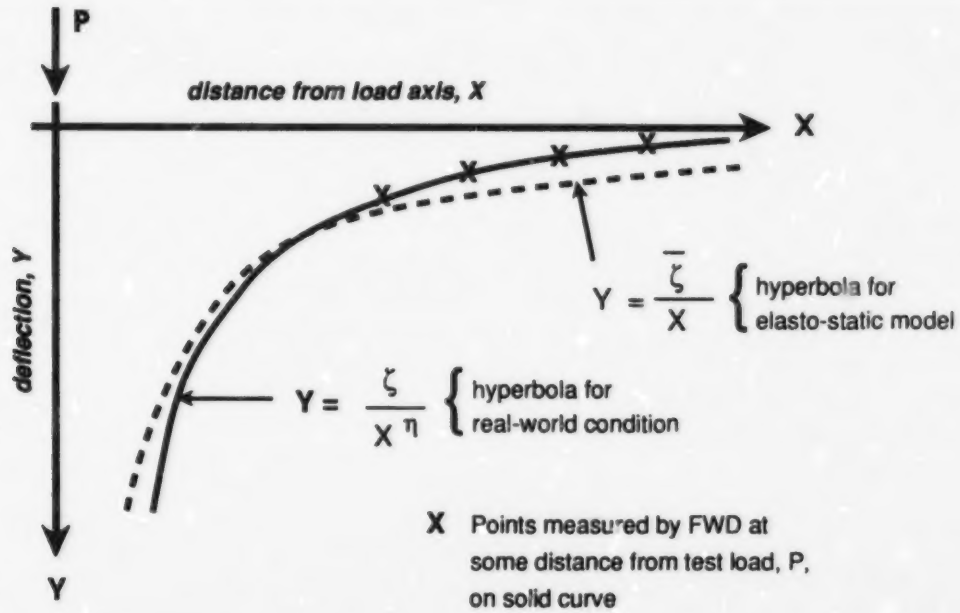


Figure 2/ Modelling of distant deflections from FWD measurements



4/ The Problem of Data Interpretation

4.1/ General Difficulties

There are two aspects to any method of non-destructive testing (NDT): 1) the mechanical functioning and reliability of the instrument, and 2) the interpretation and utilization of measurement data. Although the collection of raw data has been advanced considerably, the interpretation and use of the results in pavement evaluation work, overlay design, and management systems has lagged behind. There are reasons for this, such as the complexity and variability of pavement materials and the complexity of pavements as structural systems.

Asphaltic layers change their elastic stiffness with changes in temperature; and soil materials alter their consistency with variations in moisture, and may be subject to frost action and swelling (clay). Thus, there are seasonal, daily, and gradual long-range changes in the top layers and the substructure, not to mention the statistical variation of materials from point to point along a section.

Any non-destructive measurement of deflection is only valid for the actual time and condition it was taken. It is not valid for the next day or week, when the temperature has risen or dropped sharply. Measurements taken in early spring may not be valid for an evaluation of pavement strength the following Summer or Fall, and *vice versa*. Therefore an adjustment for temperature and for conditions in Spring are important issues.

Before and during each set of tests, air and asphalt concrete (AC) temperatures are measured. These measurements must be converted into an effective mean temperature of the asphaltic layers. This monitoring and processing of temperature is an important part of any non-destructive test method for flexible pavements.

4.2/ Efforts to Develop Interpretation Programs

In recent years much research effort has been directed toward methods of mechanistic design. In particular, many researchers have spent a great deal of work on methods of backcalculation, i.e. computing elastic stiffness properties of pavement materials using sets of measured deflections around the test load (and other information) as input. These efforts have promoted a better conceptual understanding of the mechanistic behaviour of pavement structures, although much of it has remained unclear and contradictory.

In view of these difficulties, simple methods seem to be as useful as complicated ones, because the complexity and variability of soil and asphaltic materials seem to defy accurate analysis in any case. Simple methods may even be more useful, because they are quick. When carefully modelled and programmed, they need not be, for all practical purposes, less *accurate*.

A computer program, PROBE, has been modelled and field tested in Ontario. Though fast and simple, it is not a *comprehensive* solution. It takes into account temperature (and load) adjustments, and, in its new form, the deviation of distant deflections from the Boussinesq hyperbola.

4.3/ Critical Response Parameters

Many researchers have dealt with the problem of interpretation of deflection basins [1, 2, 3, 4]. The procedure as it is currently perceived is as follows:

For a given load and contact area, and a given set of measured FWD deflections and layer thicknesses, the task is to first find the elastic stiffness values (Young's moduli) of all layers, and then other critical values such as strains, etc. With reference to Figure 3 and with regard to a standard wheel or test load, the most potent of these parameters are:

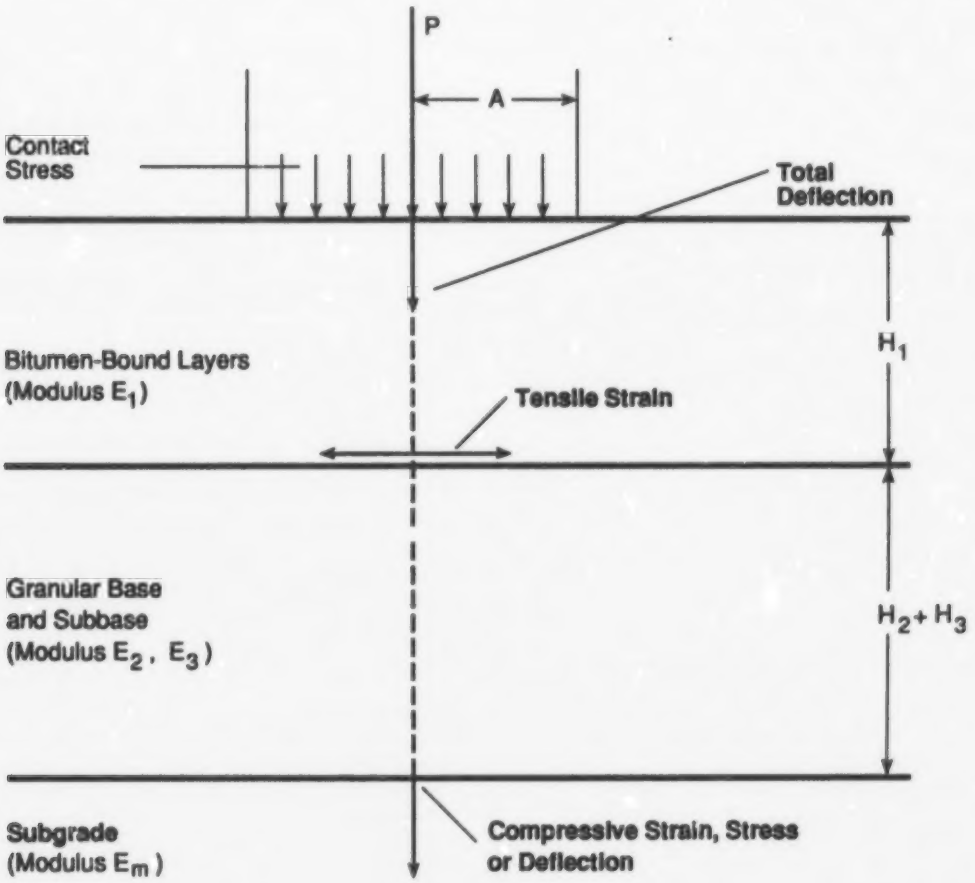
- the total vertical deflection under the load, as measured directly without data processing and reduction;
- the elastic stiffness or modulus of the subgrade;
- the horizontal tensile strain at the bottom of the asphaltic layer (AC), under the load;
- the deflection on top of the subgrade, at the load axis; and
- the compressive stress and strain on top of the subgrade, again at the load axis.

For meaningful comparisons between measurements on various pavement sections or at different times on the same section, the primary response parameters (except the subgrade modulus) must be

adjusted for the same standard AC temperature, which may be different in different climatic regions. If the target load level differs from the standard test load, then the parameters should also be adjusted to this standard load. But how can the difficulty of the very transient nature of the elastic stiffness of asphaltic layers be overcome? The traditional theory of elasticity has been used to derive solutions [1, 4, 5, 6]. But how can the gap be bridged between the assumption (of a quasi-static load acting on linear-elastic material) and the reality (of an impulse load acting on real-world materials)? For deflections at some distance from the load this gap may have a critical influence (Figure 2).

The new PROBE program addresses this problem and offers a solution that is compact and practical. Several new concepts have been incorporated in the new model, besides those which were presented earlier [5, 6].

Figure 3/ Diagram of multilayer structure



5/ The PROBE Computer Program for Real-world Pavement Material

All features of the linear-elastic solution presented earlier [6] remain unaltered (with the same good results on ELSYM5-computed deflection basins).

However, real-world pavement material is not linear-elastic and, with regard to impulse-load wave propagation, there is the assumption of a deflection basin under a static single load. These factors must be the reason the Boussinesq equation of deflection is not generally valid. Before presenting the solution to this problem, those concepts presented earlier [5, 6] that remain basically valid, shall be discussed.

5.1/ Curvature and Horizontal Strain of the AC-Layer Under the Load

The curvature and horizontal strain in the asphalt concrete layer under the load axis can be calculated directly from the measured deflections. This method has previously been discussed in great detail [5] (including comparisons with forward computations using ELSYM5 or CHEVRON programs). The bituminous layer H_1 is regarded as an elastic plate with circular symmetry of loads on top and bottom.

Figure 4 shows this plate and the geometry of the deflection bowl. Horizontal AC strain under the load is calculated by using measured deflections at and near the centre of the load.

The following equations describe curvature and strain [5]:

Curvature:

$$C = -\frac{2\left(Y_1 - Y_A\right)}{A^2} \qquad (1)$$

Strain:

$$S = -C\frac{H_1}{2}$$

or (1) and (2):

$$S = \frac{H_1\left(Y_1 - Y_A\right)}{A_2} \qquad (2)$$

where

- C = curvature under load axis (negative), in 1/mm
- S = strain at the bottom (+) or the top (-) of the asphaltic layer
- A = radius of loaded area, in mm
- H_1 = thickness of asphalt concrete layer, in mm
- Y_1 = measured deflection under the load axis, in mm
- Y_A = deflection at the edge of the loaded area, in mm.

The deflection Y_A at the edge of the loaded area is not one of the measured values; it must be calculated approximately by nonlinear interpolation, using a *bell*-curve function, as illustrated in Figure 4.

Equation 2 represents the curvature in the immediate vicinity of the load axis, within a radius equal to or less than the AC thickness. This point is illustrated in Figure 5, for an AC layer of 100 mm. To enhance accuracy, it is suggested that the second sensor (i.e., the first one outside the disk area) should be placed close to the edge of the disk; 50 mm or 2 inches is recommended.

This is a more direct way (a kinematic method) of calculating horizontal AC strains; it avoids the processing of data (such as determining the elastic stiffness of the AC layer) which is not necessary at this stage

Figure 4/ Deflection basin geometry

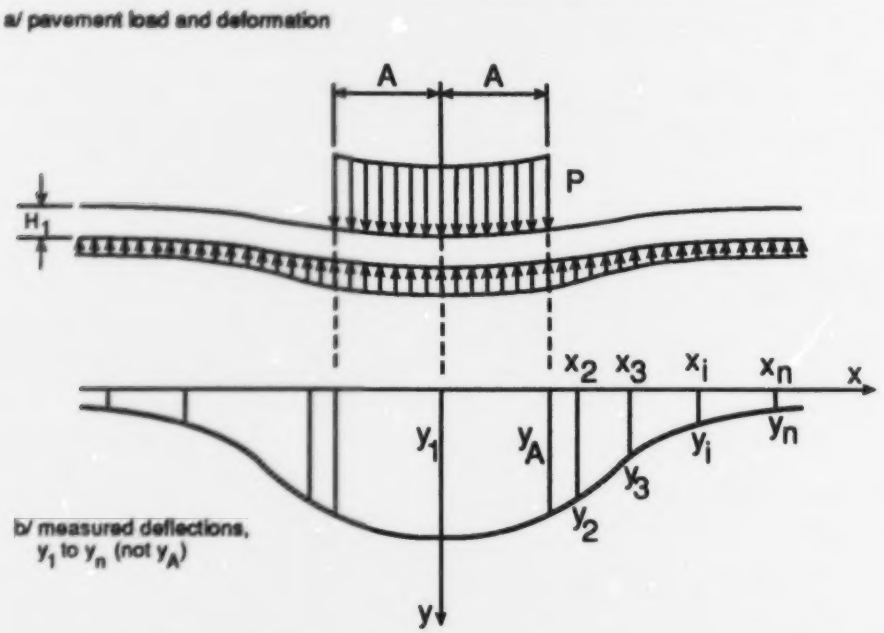
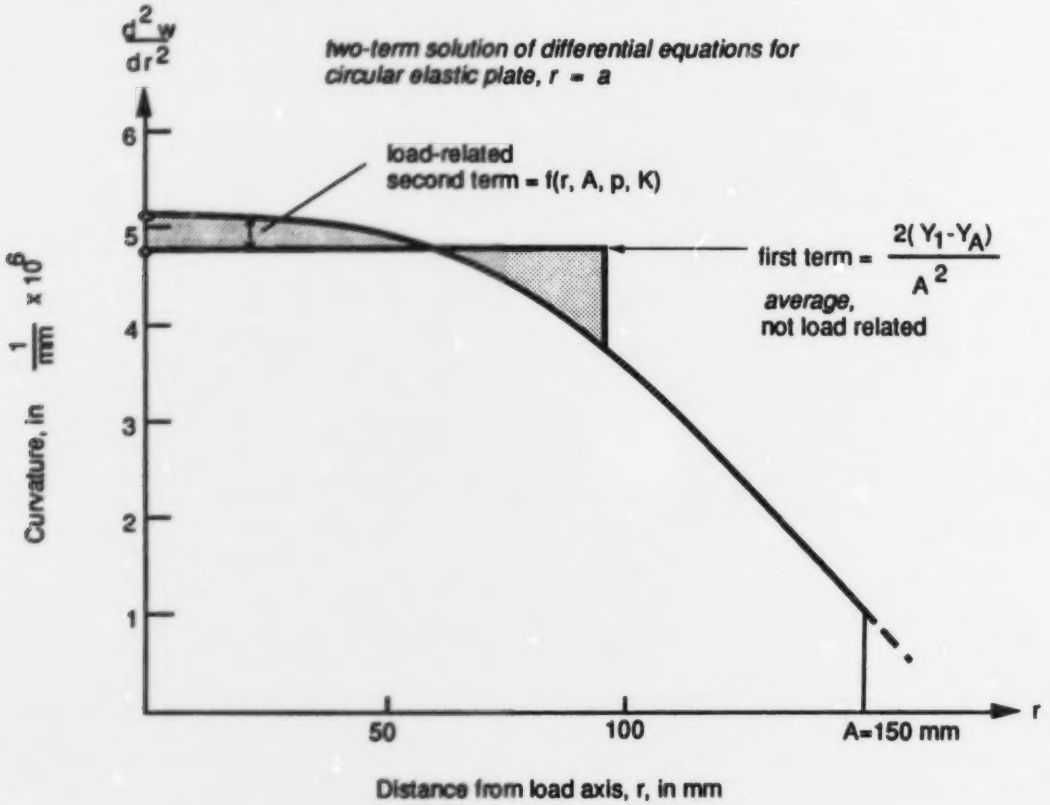


Figure 5/ Radial curvature in the asphaltic layer surface



5.2/ Temperature Adjustments

To obtain a meaningful comparison of measurements at different times or locations, all primary response parameters except the subgrade modulus must be adjusted for a (chosen) standard temperature and a standard test load. The PROBE program achieves this by computing a value for the *gradient* of the modulus E_1 with respect to the effective mean temperature of the AC layer.

The assumption is that this gradient is fairly independent of the actual magnitude of the AC modulus. However, it may change moderately with changing temperature. This would mean if there are several modulus-versus-temperature curves for various mix designs, as for instance in the Shell graph shown in Figure 6 [7], the curves would have to be parallel, but need not be straight lines.

Therefore, based on available data, an equation is derived by which a given AC modulus can be transformed into a corresponding modulus for a different temperature:

$$E_{1s} = E_{1a} \exp\left(k\,(T_a - T_s)\right) \quad (3)$$

where

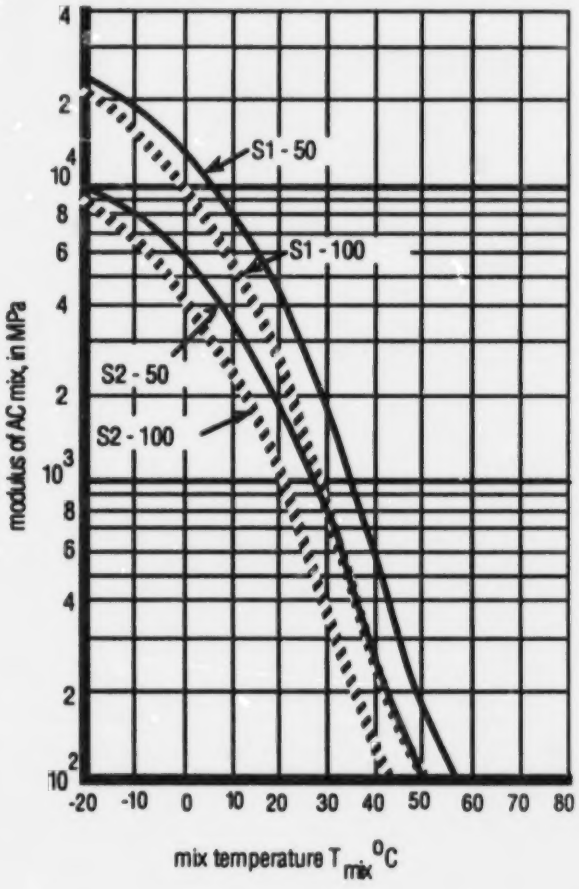
- E_{1s} = AC modulus, in MPa, at standard temperature
- E_{1a} = AC modulus, in MPa, at the effective mean AC temperature during the test
- T_a = effective mean AC temperature, in °C
- T_s = standard temperature, in °C (such as 21°C)
- k = coefficient, a function of the average temperature $(T_a+T_s)/2$.

Badu-Tweneboah *et al.* [8] in their work have implied a value of $k = 0.072$ (in their Figure 1) as being constant over a wide range. However, in order to be more accurate, the coefficient k can be modelled as a function of the average temperature $(T_a+T_s)/2$, when sufficient experimental data are available. This approach is based on the assumption that there is only a small change in the *gradient* of E_1 between T_a and T_s. In determining a k-function for PROBE, data from the aforementioned Shell graph [7] have been used (Figure 6). Of course, when modelling the program, there is also the option of curve-fitting the Shell graphs directly. It is assumed that this gradient approach of adjusting for temperature remains valid in spite of the fact that the AC modulus is also a function of loading rate.

With E_{1s} obtained from Equation 3, adjusted total deflection and adjusted subgrade parameters can be calculated by forward calculation. Thus, PROBE provides an estimate of the influence of the AC temperature on the total deflection and/or on the other primary response parameters.

If temperature adjustment is important, then FWD tests should not be carried out on very hot summer days and/or under direct sunshine. It was found that temperature adjustments of AC moduli become unreliable as temperature increases above 35°C.

Figure 6/ Shell manual, CHART M-2



5.3/ Calculation of Subgrade Elastic Stiffness (Subgrade Modulus)

According to Uuidtz [1], the elastic stiffness of the subgrade – also denoted as the *surface modulus* of the subsoil – can be calculated by the following equation:

$$E_m = \frac{P A^2 \left(1 - \mu^2\right)}{Y_R R} \qquad (4)$$

where

- E_m = elastic stiffness of the subgrade, in MPa (assuming *infinite* depth of the subgrade)
- P = tire contact pressure, in MPa
- μ = Poisson's ratio of the subgrade
- Y_R = deflection at distance R from load axis, in mm
- R = distance from the load axis, in mm
- A = radius of loaded area, in mm.

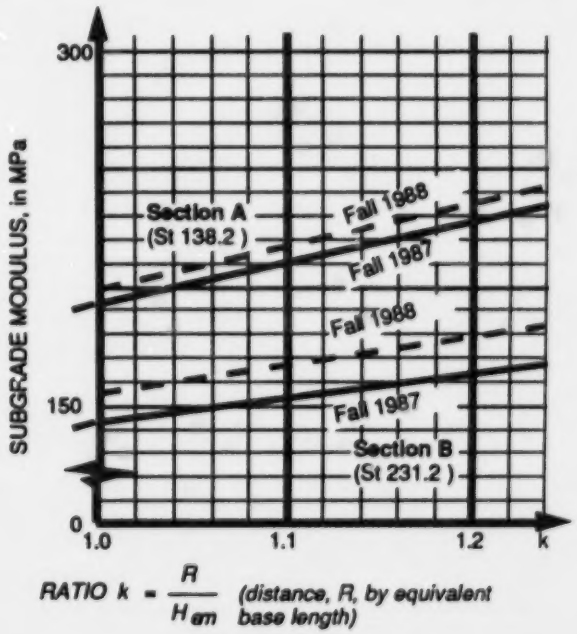
To apply this equation to pavement layers, Odemark's concept of equivalent layer thickness is used. It is said [1] that the distance R must be equal to or larger than the subgrade-based equivalent layer thickness. With this concept the pavement layers are transformed into a homogeneous half-space of the same over-all (subgrade) stiffness. Then, beyond the distance R the deflection is caused by vertical stresses and strains in the subgrade only.

Thus, the calculation of the subgrade modulus E_m in Equation 4 was assumed to be a reasonably close estimate, although it is known that the computed values from deflections on real-world flexible pavements change with R. Based on the assumption that Equation 4 is valid, the first modelling attempt was as follows [6]:
In the old PROBE program, a definite value was assigned to the distance R. It was calculated as the equivalent layer thickness H_{em}, multiplied by a calibration factor K so that $R = K^*H_{em}$, with a low value of K chosen. The results are reasonable and consistent, except for those described below, which are from a final test on raw data.

The test was done on an experimental site on Highway 7N, north of Toronto, on two adjacent pavement sections which had different layer thicknesses, but uniform subgrade conditions. Section A was of a typical strong design and Section B was much weaker in all layers. The test showed surprising results: The calculated subgrade modulus for Section A was consistently higher than that for Section B, indicating a systematic error of 20 to 35%.

The result of this test is illustrated in Figure 7. Note that, if Equation 4 were thoroughly valid, the plotted curves (here almost straight lines) should be horizontal. That they are sloped is only a minor inconsistency. A more serious inconsistency is the fact that the curves for Section A and Section B are not on the same level. The difference in level would imply that the stronger Section A has a larger subgrade modulus (by about 25%) than Section B. Disregarding small statistical variations observed by looking at all the points in each section, Sections A and B are built on the same uniform subgrade. Therefore it follows that results based on theories of static loads supported by linearly elastic materials can be affected with systematic errors of a magnitude which should not be neglected. Elimination of this error in the new PROBE program presented in this paper, is discussed in Chapter 6, refer also to earlier text pertaining to Figure 2.

Figure 7/ Subgrade modulus of adjacent sections of different strength, based on Boussinesq (old PROBE program [6])



5.4/ Backcalculation for Maximum Deflection Only

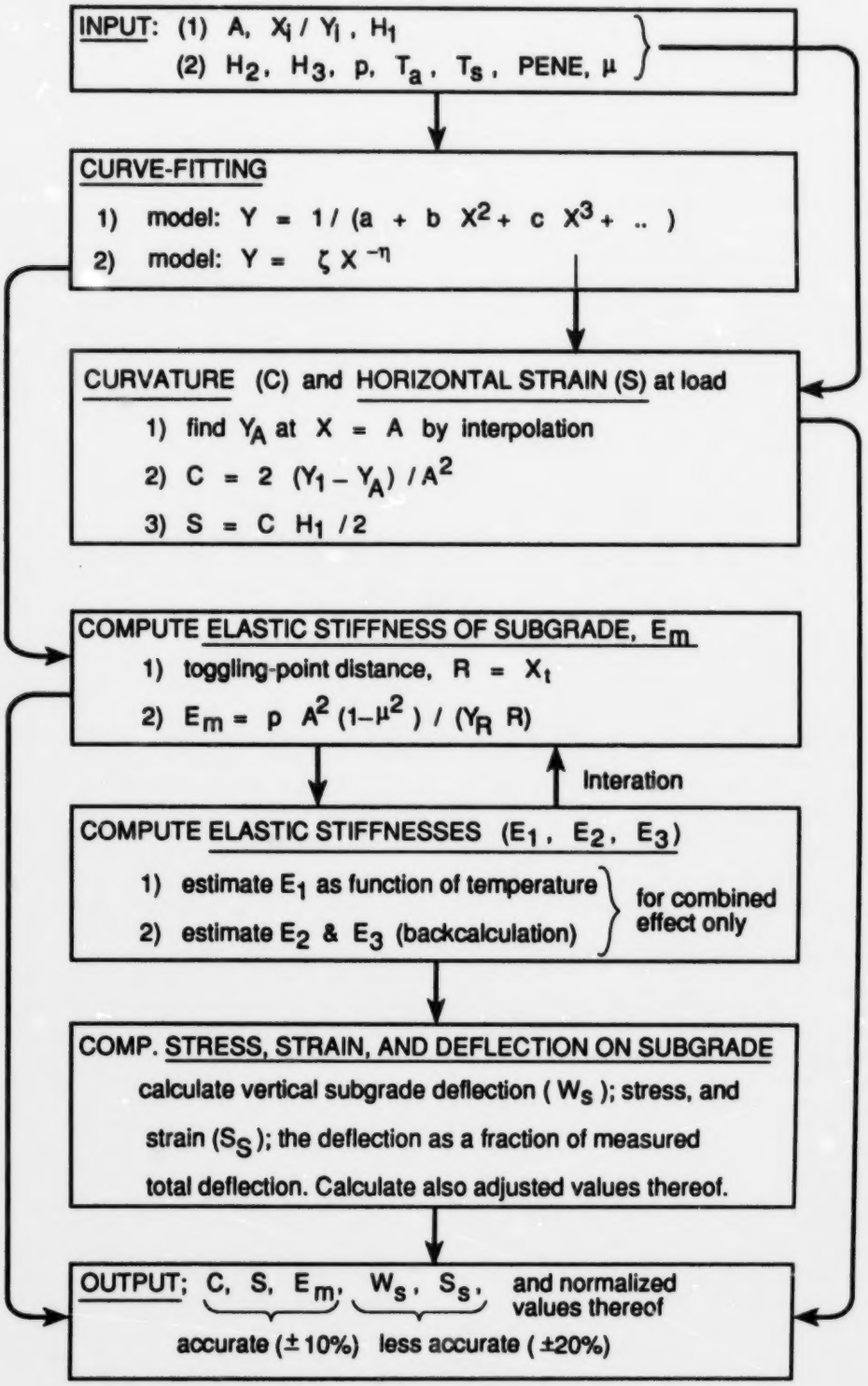
A flow diagram of the new PROBE program is presented in Figure 8. In a modified form (explained in Chapter 6) Equation 4 can still be used to calculate the subgrade modulus, as a term which changes very little within the overall iterative process. This process is based essentially on matching only the maximum deflections under the load, by comparing the measured value (from FWD measurements) with a computed value, calculated by the method of Equivalent Layer Thickness (ELT). In this iterative process the incremental changes in layer moduli can be assigned to the base and subbase layers, or to the AC layer. It is important to start with a good assessment of the quality of the AC layers and/or the granular materials underneath, in order to make reasonable estimates of those one or two layer moduli which have to be assumed and remain fixed.

The computed values of maximum deflection under the load are calculated, after each incremental change of one or two moduli, by using Odemark's method of Equivalent Layer Thickness in its original version. The method has been improved by introducing calibration functions, which calibrate the PROBE method against results obtained from corresponding calculations using elastic layer theory (ELSYM5) [6]; it reduces the systematic difference between ELT and ELSYM5 deflections to values of less than 5%. This percentage is still significant. Notwithstanding this larger percentage, it is important that the matching of deflections is carried out with a much smaller tolerance, 0.01%, because large increments in the value of moduli correspond to very small changes in deflection. Increments should decrease (but not too rapidly) with each count of the iterative steps.

What is *not* important is the accuracy of individual layer moduli values above the subgrade. It has been demonstrated that the *combined* effect of the moduli in transmitting forces to the subgrade remains almost unchanged. Even if individual moduli are changed significantly, other moduli compensate for the change, which is the effect of deflection-matching in the backcalculation process [6]. For more details on versions of the new PROBE refer to Appendix A.

For temperature adjustment, a realistic estimate of the AC layer modulus is required.

Figure 8/ PROBE Program, main branch


INPUT: (1) A, X_i / Y_i, H_1
(2) H_2, H_3, p, T_a, T_s, PENE, μ
CURVE-FITTING
1) model: $Y = 1 / (a + b X^2 + c X^3 + ..)$
2) model: $Y = \zeta X^{-\eta}$
CURVATURE (C) and HORIZONTAL STRAIN (S) at load
1) find Y_A at X = A by interpolation
2) $C = 2 (Y_1 - Y_A) / A^2$
3) $S = C H_1 / 2$
COMPUTE ELASTIC STIFFNESS OF SUBGRADE, E_m
1) toggling-point distance, $R = X_1$
2) $E_m = p A^2 (1-\mu^2) / (Y_R R)$
Interation
COMPUTE ELASTIC STIFFNESSES (E_1, E_2, E_3)
1) estimate E_1 as function of temperature
2) estimate E_2 & E_3 (backcalculation)
for combined effect only
COMP. STRESS, STRAIN, AND DEFLECTION ON SUBGRADE
calculate vertical subgrade deflection (W_s); stress, and strain (S_S); the deflection as a fraction of measured total deflection. Calculate also adjusted values thereof.
OUTPUT; C, S, E_m, W_s, S_s, and normalized values thereof
accurate (±10%) less accurate (±20%)

6/ Boussinesq Hyperbola and New Hyperbola for Distant FWD Deflections

6.1/ Some Materials Follow the Boussinesq Hyperbola

Some pavement materials follow Boussinesq. Equation 4 is based on Boussinesq's theory of the linear-elastic half-space. The deflections Y on top of the half-space around a concentrated load is a simple hyperbola with a linear term for the distance X from the load in the denominator:

$$Y = \frac{Z}{X} \qquad (5)$$

For the quasi-concentrated FWD test load the constant Z is:

$$Z = \frac{P A^2 \left(1 - \mu^2\right)}{E_m} \qquad (6)$$

A number of factorial cases were calculated, using the ELSYM5 program, to use the computed basin of deflections as input into PROBE and to compare moduli and other subgrade parameters. The computed surface deflections distant from the load axis all follow very closely a hyperbola given by Equations 5 and 6. These equations correspond to the classical theory of elasticity. For distant deflections they are also valid for linear-elastic layered systems, but only for static or quasi-static loads (not necessarily for dynamic loads).

The equations depict a hyperbola derived from Boussinesq's theory of the elastic half-space, applied to surface deflections. But the points of the FWD-measured deflections that can be expected to be located on the hyperbola, are at a distance close to or larger than the equivalent layer thickness H_{em}. Finally, note that the Boussinesq hyperbola has a *linear* term in the denominator. Unfortunately it does not adequately describe the deflections from many field measurements. This fact was also discovered by J.M. Golden for granular soil materials [9].

6.2/ Most Materials Follow a Different Hyperbola

Materials such as pavements with new bituminous layers and granular bases follow a different hyperbola [9]. Distant deflections from FWD measurements on many pavements follow closely a hyperbola with a non-linear term in the denominator (which is conceived to be toggled to a corresponding Boussinesq hyperbola):

$$Y = \frac{\zeta}{X^{\eta}} \qquad (7)$$

The constants ζ and η are dependent on layer materials, design dimensions, and load level. The exponent η can have a value close to or above unity. The constants for Equation 7 are not known as in Equation 5, *but can be computed from the measured values of distant FWD deflections by curve-fitting*. Thus, this latter hyperbola (with the non-linear term in the denominator) can always be numerically established, as long as enough distant sensors (at least three) are adequately spaced.

Examples of the two hyperbolas, Equation 5 and Equation 7, are shown in Figure 9. A measured deflection basin is compared with its concordant ELSYM5-computed deflection basin. Also being compared are the two hyperbolic curves, one pertaining to Equation 5 for the ELSYM5-basin and the other to Equation 7 for the field-data basin. Another example is shown in Figure 10. Note that, in both figures, the two hyperbolas intersect. *Only for this point of intersection are both hyperbolic equations (5 and 7) equally valid.*

For modelling PROBE, the Boussinesq theory has been used (Equations 4, 5, and 6). However, distant FWD deflections follow the new Equation 7. Thus, it is necessary to go back along the curve of Equation 7 to its point of intersection with the corresponding curve of Equation 5. It is only there that a valid pair of the needed values R and Y_R can be found for Equation 4, to calculate the subgrade modulus.

Figure 9/ Hyperbolas and deflection curves for elasto static model and real-world material

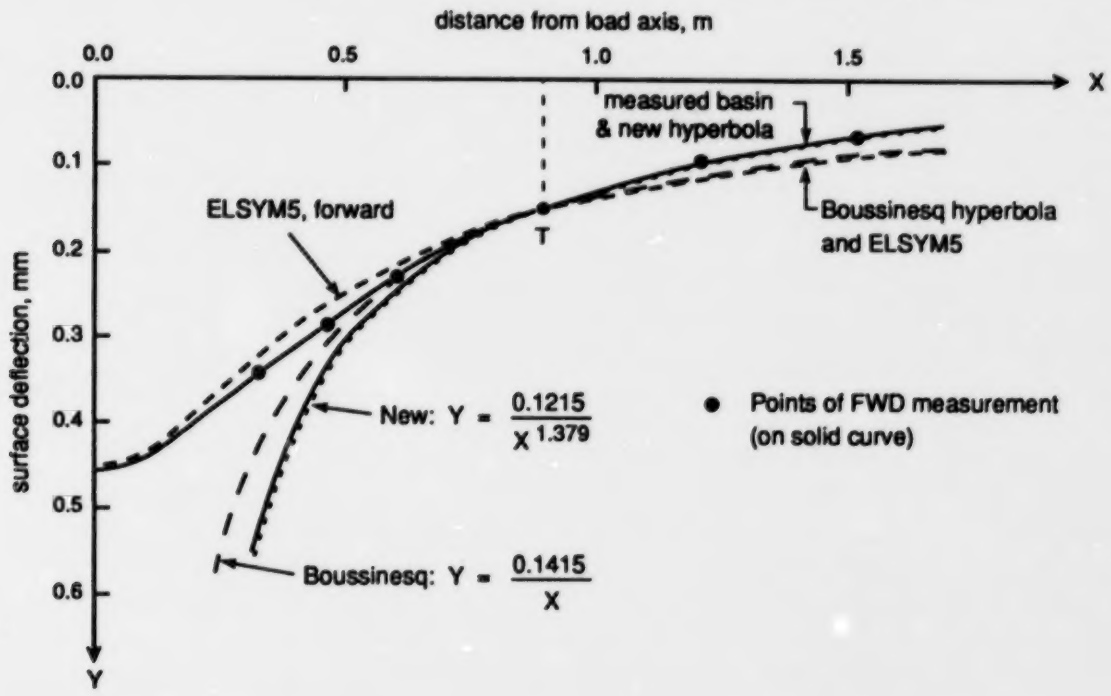
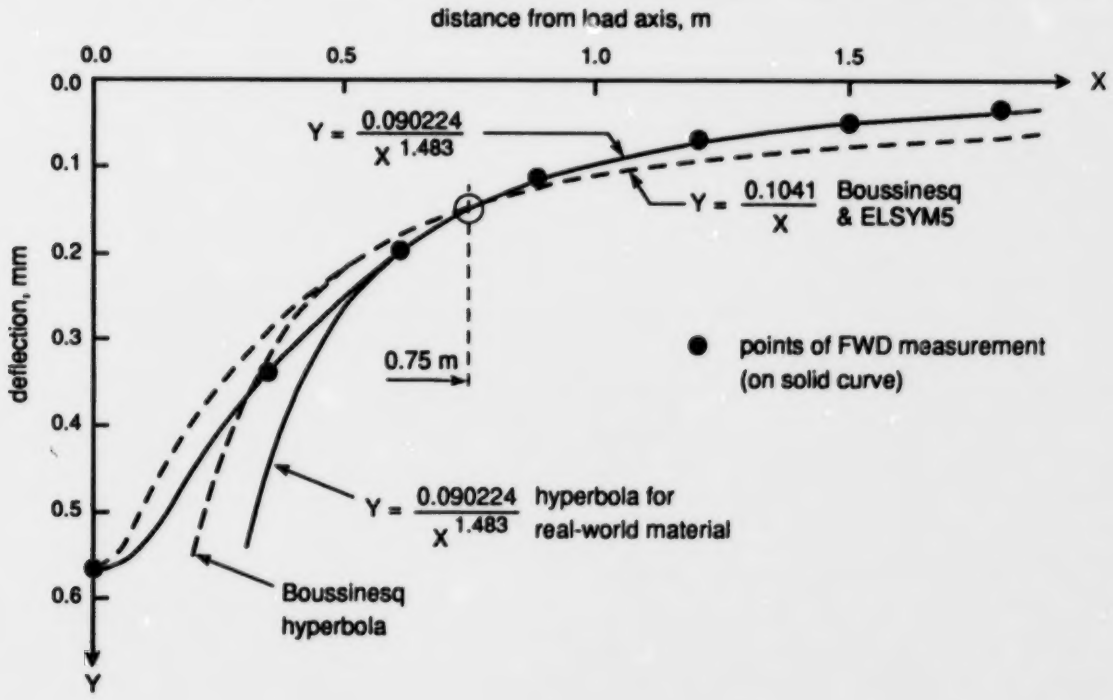


Figure 10/ Example, measured deflection basin (solid), and corresponding ELSYM5 deflections with Boussinesq hyperbola (dotted)



6.3/ Modelling PROBE for Real-world Pavement Materials

Modelling of the PROBE program consists of two tasks. First, note that the Boussinesq-based Equations 5 and 6 can be regarded as a special case of the new Equation 7. Thus, the first task is a careful modelling and calibration along the hyperbola of Boussinesq. This was done by using deflections from ELSYM5-computed cases as input and ELSYM5 results as bench-mark. In this part of the modelling the results of computations are independent from the ratio $K = R/E_m$, i.e., lines such as in Figure 7 would be horizontal. This Boussinesq side of the solution has been improved slightly from the earlier one [6] by hyperbolic curve-fitting along Equation 5 (or Equation 7, with $\eta = 1$).

The new version of PROBE presented here, has been tested on a number of cases computed with the ELSYM5 program as before [6], with the following results. The subgrade moduli resulting from the PROBE runs match those of the ELSYM5 input within –3 and +0 %, for a large range of practical design dimensions. The small deviations are strictly a matter of the accuracy of both, the ELSYM5 calculations and the hyperbolic curve-fitting (by Equation 5 in this case). In other words, the Boussinesq side of the solution is now theoretically correct. However, ELSYM5 is only the bench-mark for the more general solution, checking only one side or aspect of it. It is still necessary to develop the general solution with $\eta > 1$.

The second task is to curve-fit the measured points near and beyond the radius H_{em}, to a hyperbola with a non-linear term in the denominator, by using Equation 7. Once established, this hyperbola is then pursued (i.e., extrapolated) back toward the load, to the point of intersection with the corresponding curve of Equation 5 (Boussinesq). This latter curve always exists and pertains to an ELSYM5 case concordant with the field case computed by PROBE. The concordance is based on matching maximum deflections. This means an ELSYM5 case can be created which shares with the PROBE case the values for layer thicknesses and sets of values of layer moduli. Both cases have the same maximum deflection under the load (by definition), and the test load is assumed to exert the same effect on the subgrade. (Note that individual values of moduli need not be equal; refer to the theorem of invariability of total moduli effect on subgrade parameters [6], or Chapter 7.)

Although this concordant ELSYM5 case could be established for any particular field case, it is generally not feasible to do so. However, all that is needed is the point of intersection which can be found by a process of calibration, directly based on field data. For the Highway 7N test sites (A and B) the point was estimated to be at a distance of about X_t = 0.75 to 0.80 m from the load axis. It should be noted that small differences in this estimate have little influence, because the curves intersect at an obtuse angle (Figures 9 and 10). Indications are that this value is also a fairly good estimate for other cases of similar design (i.e., AC layer on granular bases).

Table 1 illustrates how the new PROBE program works on the two adjacent Sections, A and B, on Highway 7N. The new version of PROBE has removed the inconsistency shown in Figure 7 with regard to the aforementioned systematic error in computed subgrade moduli for Sections A and B.

Table 1/ Subgrade modulus for two adjacent sections X_t = .75 m

Section A (Fall 1988 data)		Section B	
Point	Modulus	Point	Modulus
9	168.8 MPa	0	163.1 MPa
8	156.5 MPa	1	160.9 MPa
7	178.8 MPa	2	152.0 MPa
6	138.6 MPa	3	166.5 MPa
Average:	160.7 MPa	*Average:*	160.6 MPa

For a similar test on the same section with FWD data from a different year and season, refer to Section 7.3.

The pavement in Sections A and B consists of asphaltic layers with a dense friction course, a Granular-A base, a sand subbase, and a silty clay subgrade, all of uniform quality.
The layer thicknesses were (in mm):
Section A: 125 AC, 165 Gr. A, 520 sand (approx.);
Section B: 90 AC, 155 Gr. A, 410 sand (approx.).

The new concept of subgrade modulus calculation is illustrated by an example in the following section.

6.4/ Example

A point in Lane 1 of Section B was selected with the following FWD data (Fall 1986, measured at 16°C, A = 150 mm):

Layer thicknesses: 90 mm AC
145 mm Gr. A base
390 mm subbase

Measured FWD deflections (in mm):

X:	0	300	600	900	200	1500	1800
Y:	0.526	0.352	0.189	0.107	0.068	0.049	0.038

The concordant ELSYM5 case has the following computed deflections (from forward-calculation with ELSYM5, using PROBE output data):

Y:	0.5278	0.3078	0.1724	0.1160	0.0870	0.0695	0.0578

Note that the maximum deflections match with a deviation of 1.8 µm only. However, the deflection basins follow different curves (Figure 10). The results of PROBE calculations are:

Subgrade modulus: 143.52 MPa

Subgrade deflection: 0.2260 mm

(forward ELSYM5: 0.2264 mm)

The new version of PROBE also calculates the curve-fitted values of Equation 7 (from Y_2 to Y_7), to provide guidance for the user. The two deflection basins, their corresponding hyperbolas, and the toggling point of intersection are shown in Figure 10. Note that the point of intersection, where the Boussinesq hyperbola and Equation 7 are valid, is at about 0.75 m.

7/ Experimental Consistency Checks on Field Data

There are certain consistency checks which the PROBE program (as any other program) should pass. For the the original PROBE program, such tests have been presented [6]. All tables in this chapter (as well as Table 1) are computed by the new PROBE program.

7.1/ Subgrade Modulus for Different Target Levels of Test Load

The first requirement of consistency is that, with increasing target level of test load, the subgrade modulus should not change or change only slightly at the same test location. A slight change is to be expected. Unless there is much scattering in the FWD measurements, the modulus should decrease and not increase with increasing test load, which is natural for all materials that are not ideally elastic. This important point is shown in Table 2.

Table 2/ Subgrade moduli for different test load levels

Sensor distances, in m: 0.6, 1.0, 1.5, 2.0; for Fall 1988
0.9, 1.2, 1.5, 1.8; for Fall 1986

Section B Hwy 7N	Fall 1988	Fall 1986
Loads, kPa	Modulus, MPa	Modulus, MPa
35.5 / 34.1	204.3	157.1
44.3 / 43.4	182.8	150.2
54.1 / 54.0	180.5	147.9
65.2 / 63.6	197.7	143.1

Note that the far sensor distances chosen in 1986 lead to marginally better results (less scattering, range 9% for 1986 versus 13% for 1988). The values in Table 2 are typical for other tests.

7.2/ Combined Effect of Upper Layer Moduli

Another important requirement is for the combined effect of upper layer moduli on the subgrade to remain unchanged, even when the modulus of the AC layer is estimated with a very large error. This theorem of the invariability of the combined effect on the subgrade of any set of upper layer moduli, first presented by Jung [6], can be demonstrated to be true for the new version of PROBE, (Table 3):

Table 3/ Combined effect of upper layer moduli (I)

Subgrade parameters computed over the *same* deflection basin.
Maximum deflection: 0.5620 mm, Load: 54.36 kN,
A = 150 mm.
Test load and deflections: as in the example, Figure 10.*

Asphaltic (AC) layers estimated to be:	softer		medium	harder	
Assumed AC modulus, MPa	1000	2000	4000	6000	8000
Base modulus, MPa *backcalculated*	513.7	427.6	345.4	302.0	279.5
Subbase modulus, MPa	273.6	227.7	183.9	160.8	148.8
Subgrade modulus, MPa			143.9		*
Vertical deflection on top of the subgrade, mm	.2287	.2283	.2259	.2235	.2214
Vertical strain on top of the subgrade (micro)	332	330	321	312	304
Vertical stress on top of the subgrade, kPa	47.8	47.5	46.2	44.9	43.7

*Note: This result differs from that of the example because an earlier version of PROBE was used.

Since the calculations used the same measured FWD deflections, the subgrade modulus remains constant. More importantly there is very little

change in the other subgrade parameters. Table 3 is typical for many other test results, and the theorem may be generally true with regard to other backcalculation methods. Another example is presented in Table 4, a case of deteriorated AC layer, with raw data from a test site in Alberta.

Table 4/ Combined effect of upper layer moduli (II)

Subgrade parameters computed over the *same* deflection basin.
Maximum deflection: 0.4500 mm, Load: 39.23 kN,
A = 150 mm.
Test load and deflections: as in the example, Figure 9.

Asphaltic (AC) layers estimated to be:	**very soft or broken**			**medium**	
Assumed AC modulus, MPa	500	1000	2500	3000	5000
Base modulus, MPa (calc.) *backcalculated*	523.4	260.7	122.7	115.0	N/A
Subbase modulus, MPa	-	-		-	-
Subgrade modulus, MPa		**79.25**		79.31	N/A
Vertical deflection on top of the subgrade, mm	.2899	.3041	.2957	.3053	N/A
Vertical strain on top of the subgrade (micro)	350	389	366	358	N/A
Vertical stress on top of the subgrade, kPa	27.8	30.8	29.0	26.2	N/A

7.3/ Test of Toggling-Point Distance on Section A and B of Highway 7N

The FWD deflections measured at a distance of about $0.75*H_{em}$ or larger from the load axis can be curve-fitted along a hyperbola as represented by Equation 7. For modelling the calculation of subgrade moduli, it is necessary to use Equation 4, which is based on Boussinesq's theory of elastic half-space. Deflections distant from the load axis calculated by ELSYM5 or similar programs follow Boussinesq's theory, Equations 5 and 6. For each measured FWD deflection basin under an impulse test load, a corresponding ELSYM5 case can be constructed with the same maximum deflection in the centre, as illustrated by the example (Figure 10). Both, the real-world case as measured by FWD and the ELSYM5-constructed case, have their deflection basin curve with the particular hyperbola at the tail (Figure 10). The only point where a valid pair of distance and deflection values can be obtained (to be used in Equation 4) is at the intersection of the two hyperbolas, the toggling-point, X_t. The new, more general hyperbola (Equation 7) can be established numerically by curve-fitting the measured FWD deflections. Its toggling-point to the Boussinesq hyperbola must be found or verified by calibration.

The previous example in conjunction with Figure 10 illustrates calibration for a single instance, working with a concordant ELSYM5-computed case. On a larger scale, the test sections, A and B, can be used to verify the toggling-point distance, as both sections are constructed of uniform material but different design thicknesses. Adjacent points of the sections should have the same subgrade modulus, except for some statistical scattering. For the first time both sections were tested together by FWD, on the same day in the Fall of 1987, at an effective mean AC temperature of 8°C. The sensors were spaced evenly at a distance of 300 mm. Target load levels 3 and 2 (55 kN and 44 kN) were chosen for Sections A and B, respectively, in correspondence with their relative strength.

Table 5/ Verification of subgrade modulus and toggling-point distance on two adjacent test sections on Highway 7N

Section A			Section B		
	Subgrade Moduli			Subgrade Moduli	
for			for		
X_t =	0.75 m	0.825 m	X_t =	0.75 m	0.825 m
Point	MPa	MPa	Point	MPa	MPa
9	135.27	151.87	0	141.34	157.45
8	135.07	150.70	1	147.00	156.23
7	161.35	178.19	2	147.86	159.47
6	134.09	149.14	3	139.63	148.94
5	130.95	145.66	4	169.78	178.16
			5	130.78	137.68
Averages:	**139.35**	**155.11**		**146.07**	**156.32**

Table 5 shows the results for two assumed toggling-point distances, 0.75 m and 0.825 m. In both cases the corresponding averages from points on Section A and Section B are reasonably close. The larger distance of 0.825 m has lead to a result which is marginally better. This was not the case with the Fall 1988 data used in Table 1.

It should also be noted that the subgrade modulus increases with increasing toggling-point distance. Thus, calibration is important, and results cannot be expected to be very accurate. The results are also affected by the number and distribution of the distant measuring points which are used for the curve-fitting procedure. More experience is needed with the new PROBE program.

In 1989 similar results were obtained on a test section of Highway 80, near Sarnia, Ontario. FWD tests were carried out before and after a two-lift overlay. The toggling point distance was confirmed to be X_t = 0.75 m.

8/ Sensitivity Analysis for Subgrade Modulus

After curve-fitting the tail of the deflection basin by dint of Equation 7 and for a given distance of the toggling-point, the subgrade modulus can be expressed by the following equation (omitting a calibration factor):

$$E_m = \frac{P A^2 \left(1 - \mu^2\right) X_t^{\eta-1}}{\zeta} \qquad (8)$$

where

E_m = subgrade modulus

A = radius of load distributing disk

μ = Poisson's ratio

X_t = distance of toggling-point from load axis

η, ζ = constants found by curve-fitting procedure

The following is apparent:

a) The subgrade modulus is insensitive to errors with regard to the depth of the subgrade surface. This depth (which is equal to the sum of the layer thicknesses) is only important for determining the start of the basin tail for curve-fitting, i.e., for deciding which sensor points to include in the curve-fitting process and which to exclude. For this decision, only a very approximate estimate of the depth (or total upper layer thickness) is needed.

b) Differential calculus on Equation 8 leads to the following relationship of error percentages:

$$(\%\ \text{error of}\ E_m) = (\eta - 1) * (\%\ \text{error of}\ X_t) \qquad (9)$$

For instance, for a value of $\eta = 1.5$, a 10% error in the toggling-point distance would result in a 5% error of the subgrade modulus.

Note that the signs are positive, i.e., an increase in X_t leads to an increase in E_m, as shown also in Table 5. Further note that for materials ($\eta = 1$) which follow the Boussinesq equation, there is no error in E_m from an error in X_t, i.e., any distance X_t will do, since the measured points are now curve-fitted by the Boussinesq hyperbola.

8.1/ Selection of Sensor Locations for Hyperbolic Curve-fitting

The measured FWD deflections must be curve-fitted by dint of the hyperbolic Equation 7. Only the *distant* sensor points are curve-fitted. Those points which are too close to the load are excluded. The definition of *too close* is: *the distance at which the deflection basin deviates from the hyperbola, i.e., the point at which the measured deflection is distinctly smaller than the corresponding hyperbola value.* By theoretical expectation, this must be the case when the distance of the sensor is *well within* a radius equal to the subgrade-based equivalent layer thickness. In practical terms, the first two to four sensors of a total of seven may be excluded. The phrase *well within* means that the equivalent layer thickness, H_{em}, is not a rigid cut-off point. All this can be verified by studying curve-fitting errors from actual field data.

The optimal cut-off distance around the load axis is difficult to estimate. Using only the last two points on the curve would not render an optimal solution because deflections are very small. Five points may also be inaccurate, because the closest point may deviate too much, so that the advantage of curve-fitting a larger number of points disappears. The example, Figure 10, was based on four points, which is normal. With five points the results would have been very similar, but only in this case, not in other cases. Normally, there should be four sensor readings available for curve-fitting.

It might be appropriate to increase the number of sensors from seven to eight (A = 150 mm), namely at:

1) 0, 200, 400, 600, 900, 1200, 1500, 1800 mm, for medium pavements; and

2) 0, 200, 500, 800, 1100, 1400, 1700, 2000 mm, for heavy pavements.

This would improve the non-linear elastic solution but would also increase the cost.

8.2/ Testing of Cases from ELSYM5-Computed Deflections

A set of tests, similar to those depicted in Tables 1 to 6 in Jung [6], were carried out with the new version of PROBE. The deflection input data (output from the ELSYM5 cases) were curve-fitted accurately by Equation 7 with $\eta = 1$. The results were more accurate and exhibited a larger degree of consistency than those computed earlier [6], because of the error-reducing effect of curve-fitting several deflections as opposed to picking only one deflection value. The new tables are not presented, because the results are as expected, based on an understanding of Equations 8 and 9. In these tests the deviations between the PROBE and the ELSYM5 values of subgrade moduli were between 0 and -3%; i.e., the computed PROBE values were slightly lower than (or equal to) the ELSYM5 input values of E_m. In order to assure these conservative results, some minor calibration was needed for heavy pavements with thick granular subbase or base layers.

9/ Summarizing Achievements and Further Research

The new version of the PROBE program is an improvement over the original one presented earlier [6]. It has been upgraded to provide a more general, but still simple solution. The distant FWD deflections measured on real layer materials follow a more general hyperbolic curve (Equation 7) of which Boussinesq's hyperbola (Equation 5) is only a special case, valid only for certain materials. Consistent results can still be obtained by Odemark's ELT method; following the new, more general hyperbola to the toggling-point at a relatively fixed distance, X_t, from the load axis. This distance has been approximately determined for normal Ontario designs, i.e., for asphaltic layers on granular base and subbase. A greater variety of cases must be tested, including different base materials, such as stabilized and/or treated bases.

Adjustment for normal temperature has been achieved for each set of FWD deflection data.

Interpretation is still weak with regard to adjustments for the weakness of subgrade in Spring, and research in this area would be most useful.

10/ Conclusions and Recommendations

The original PROBE computer program [6] has been modified to take into account the *power law* of surface deflections first discovered by Golden [9].

As before, the Boussinesq hyperbola of surface deflections is used for calculating the subgrade modulus, however, the deflection Y_R is taken at a certain fixed distance $R = X_t$ from the load axis which is conceived as the toggling-point of two hyperbolas.

It is shown that, in general, deflections measured by FWD, at some distance from the load, follow a different hyperbola (with a non-linear term in the denominator) which has one point in common with a concordant Boussinesq hyperbola. The coordinates of this point are assumed to be valid input for Equation 4 (based on Boussinesq), for calculating the subgrade modulus. But these coordinates must be taken from the new hyperbola, to be numerically established by curve-fitting the distant FWD deflection data. At the Highway 7N test site near Toronto, this toggling-point of the two hyperbolas was found experimentally, to be approximately $X_t = 0.75$ to 0.80 m, by comparing adjacent sections of pavement with different design strengths.

Some research should be directed to verify this concept for different kinds of layer materials. In particular, the toggling-point distance X_t could change slightly for different base materials and for a different radius of load distribution.

Recommendations for FWD sensor distances:

1) The last four sensors should be well distributed over the tail of the deflection basin, between a distance X from about 0.8 to 2.0 times the equivalent layer thickness, H_{em}.
2) For a disk radius of $A = 150$ mm, the first three sensors should be at $X = 0$, 200, and 400 mm, the second sensor being close to the edge of the disk.

Appendix A/
Versions of PROBE Program and Practical Application

A-1/ Input and Output

Input for all versions of PROBE must be provided on files stored on disks. A small program (in BASIC) creates the files in the appropriate format (PRO-FILE.BAS). Input files can be updated or amended by another program (P-FILE-M.BAS). Besides the seven measured deflections and the corresponding six sensor distances, several other pieces of information are required, such as layer thicknesses, contact pressure (load), radius of load distribution, average AC temperature, etc. The user is guided by computer-prompted questions and is also asked to specify an output file.

Output is printed directly on screen and simultaneously to an output file. The output starts with an echo-print of input data and includes guidance for evaluating the hyperbolic curve-fitting of the distant measured deflections. For example, a value of η equal to or close to 1 would mean that the pavement material behaves almost in accordance with Boussinesq's half-space theory and, in such a case, the choice of the toggling-point distance would have no influence on the results. Further, a direct comparison of measured and curve-fitted distant deflections allows an evaluation of the quality of curve-fitting and the quality of measured deflection data, including their adequate distribution and stretch.

The output values adjusted for standard temperature and load are printed behind the unadjusted values (which are directly based on the measured deflections). There is no adjustment for the subgrade modulus. The printed layer moduli are the estimates used by the program and are possibly incorrect (i.e., correct only with regard to their combined effect on the subgrade).

A-2/ Versions of the PROBE Program

As before [6], the versions of the new PROBE program are also written in BASICA or GWBASIC and guide the user by prompting in an interactive mode. There are two versions:

1) *PROBE.BAS*

   This program is written for asphalt pavements (AC) with one or two granular base layers, for a large variety of dimensions. In order to compute an adjustment for normal temperature, an estimate of the elastic stiffness of the AC layer is included (based on the quality of mix design and of the AC temperature at the time of testing [7]). For cracked or otherwise ill-defined AC layers, PROBE has a branch in which the base modulus is estimated and fixed and the AC modulus E_1 is backcalculated.

2) *PROBEX.BAS*

   This is a universal program which combines the structural layers above the subgrade into one layer. Adjustments for temperature are made only in case of full-depth AC designs. For the usual layered structures (AC, base, subbase), no adjustments for normal temperature are possible. On the other hand, the program is suitable for designs with lime- or cement-treated bases and for layer structures with somewhat uncertain dimensions where temperature adjustments are not required or not possible. However, an estimate of the subgrade depth or total layer thickness ($Z = H_1 + H_2 + H_3$) is required in order to compute subgrade parameters (other than the subgrade modulus). The backcalculation via maximum deflection was modelled after Kirk [10].

Both programs compute the radius of surface curvature under the test load and the AC tensile strain, which is only valid for a non-broken, non-laminated first (AC) layer, supported by granular or other soil material.

A-3/ Examples

PROBEX:
The adjusted values are for standard load (9000 lb or 40 kN) only. Adjustment for a standard normal temperature (21°C) in PROBEX is not feasible except for full-depth asphalt concrete designs.

PROBE* Analysis , *PROBEX

For full-depth asphalt pavement on subgrade (without base), or for structural layers combined into one layer (comb.layer modulus). In this case there in no adjustment for temperature (only for load).

Load, in kN:	54.357
Radius, in mm:	150
Contact pressure, in kPa:	769
Layer thicknesses, in mm:	90, 145, 390, 0 (subgrade)
Modification factor for subgrade modulus:	1.0000 (< 1, in case of finite subgrade layer)
Temperatures, at test (and adjusted), in °Celsius:	21 (21)
Original penetration of the asphalt:	88
Type of asphalt concrete:	S-1

Primary Response parameters:	**Unadjusted**	**Adjusted**
Total deflection, in mm:	0.52600	0.38740
Radius of AC curvature, in m:	186.4	253.0
AC tensile micro-strain:	241.5	177.8
Subgrade deflection, mm:	0.23476	0.17290
Subgrade stress, in kPa:	48.54	35.75
Subgrade micro-strain:	338.2	249.1
Comb.layer modulus, in MPa:	451.2	451.2

η, ζ: 1.4830610, 0.0902239, J1 = 4
X(J1), H_{em}, in metres: 0.90000 0.82400

Subgrade modulus, in MPa: 143.518, 143.518, (modified)

Sensor	Distance, m	Deflection, mm	Hyperbola, mm	Error of Fit, %
1	0.000000	0.526000	0.000000	0.000000
2	0.300000	0.352000	0.538001	52.841080
3	0.600000	0.189000	0.192458	1.829863
4	0.900000	0.107000	0.105483	-1.417664
5	1.200000	0.068000	0.068848	1.246997
6	1.500000	0.049000	0.049450	0.918695
7	1.800000	0.038000	0.037734	-0.699021

The following is a normal PROBE run for relatively new asphaltic pavements. It corresponds to the example illustrated in Figure 10.

Output file of *PROBE* Run

For up to three layers above subgrade (AC, base, subbase).

Load, in kN:	54.357
Radius, in mm:	150
Contact pressure, in kPa:	769
Layer thicknesses, in mm:	90, 14, 390, 0 (subgrade)
Temperatures, at test (and adjusted), in °Celsius:	16 (21)
Original penetration of asphalt:	88
Type of AC/modification factor for subgrade modulus :	S- 1 / 1 (< 1, for finite subgrade layer)
(AC modulus, unadjusted and adjusted, in MPa:	4210.6, 2820.7,)

Primary Response Parameters:	**Unadjusted**	**Adjusted**
Total deflection, in mm:	0.52600	0.41618
Radius of AC curvature, in m:	186.4	235.5
AC tensile micro-strain:	241.5	191.1
Subgrade deflection, mm:	0.22599	0.17108
Subgrade stress, in kPa:	45.95	34.78
Subgrade micro-strain:	320.1	242.3

η, ζ: 1.4830610, 0.0902239, J1 = 4
X(J1), H_{em}, in metres: 0.90000 0.80289

Subgrade modulus, in MPa: 143.518, 143.518, (modified)

Moduli sequence, in MPa : 4210.6, 341.5, 181.9, 143.5,

Sensor	Distance, m	Deflection, mm	Hyperbola, mm	Error of Fit, %
1	0.000000	0.526000	0.000000	0.000000
2	0.300000	0.352000	0.538001	52.841130
3	0.600000	0.189000	0.192458	1.829871
4	0.900000	0.107000	0.105483	-1.417637
5	1.200000	0.068000	0.068848	1.247008
6	1.500000	0.049000	0.049450	0.918695
7	1.800000	0.038000	0.037734	-0.699021

A-4/ Running PROBEX

In running the PROBEX program, the user is asked to select the first sensor point. It can quickly be computed, checked, and reselected. More critical in PROBEX is the fact that the user must select a seed value for the combined modulus E_1 of the structural layers. If the user's selection is too far off, it may lengthen the time to obtain the results. In this case it may be faster to start again and select an entirely different value, etc. If the selection is reasonably close, the results typically are printed in less than ten seconds on the slowest MS-DOS computer.

The user is also asked questions about design features inorder to input some minor calibration of the subgrade modulus and the subgrade deflection. These calibrations were derived from comparative ELSYM5 calculations, using ELSYM5 deflections as input [6]. In the strictest sense, the calibrations are valid for the *Boussinesq* side of the solution (with $\eta = 1$). It was found (by studying η values) that some designs of flexible pavements behave nearly in accordance with Boussinesq. These were pavements very old AC layers.

The PROBEX program works on all the cases that were presented by R. Lytton in January 1988, at TRB. In calculating these cases, the cement- and lime-treated bases were regarded as the top layer H_1, E_1 and the AC layer merely as a load-distributing pad.

A-5/ Program for Overlay Design

PROBE has been expanded to include a process of point-by-point overlay design, to be used for chain-plotting a required minimum overlay thickness for each station or point of FWD measurement. The total thickness of the overlay lifts can then be selected as a value along or near the peaks of the chain plot. Recycling by milling off part of the old asphaltic layers can be taken into account by subtracting a corresponding value from the old AC layer thickness. The required minimum overlay thickness pertains then to the thickness of the new mix which usually contains a certain percentage of recycled material. The method is dependent on a standard design value of deflection at the load axis, adjusted or normalized for a standard test load (9000 lb or 40 kN) and a standard temperature (such as 21°C).

Standard design values of normalized FWD deflections under the test load are a function of road class, heavy vehicle traffic volume, and regional climatic conditions. They can also be regarded as related to the life span of the road section within the context of a pavement management system.

The overlay brings the adjusted maximum FWD deflection down to an acceptable value for that particular road class, after it has grown beyond that value due to the deterioration of the structural layers.

The output of the expanded PROBE program contains an echo-print of the interactive input decisions and estimates of improved subgrade parameters, i.e., reduced values of adjusted vertical strain, stress, and subgrade deflection, adjusted for normal temperature and standard load. The following examples show the same required overlay thickness (as it should be) for two different levels of test load. Similarly, because the overlay design is based on temperature adjusted values of AC moduli, the air or AC temperature during the test should not influence the result.

Example 1

Output file of *PROBE* Run
For up to three layers above subgrade (AC, base, subbase)

Load, in kN:	43.542
Radius, in mm:	150
Contact pressure, in kPa:	616.0001
Layer thicknesses, in mm:	90 , 145 , 390 , 0 (subgrade)
Temperatures, at test (and adjusted), in ° Celsius:	16 (21)
Original penetration of asphalt:	88
Type of AC/modification factor for subgrade modulus :	S-1 / 1
(AC modulus, non-adjusted and adjusted, in MPa:	4210.6, 2820.7,)

Primary Response Parameters:	**Unadjusted**	**Adjusted**
Total deflection, in mm:	0.42500	0.42031
Radius of AC curvature, in m:	218.2	220.6
AC tensile micro-strain:	206.2	204.0
Subgrade deflection, mm:	0.17462	0.16506
Subgrade stress, in kPa:	38.65	36.53
Subgrade micro-strain:	255.2	241.2

η, ζ: 1.3401100, 0.0713615, J1 = 4
X(J1), H_{em}, in metres: 0.90000 0.78473

Subgrade modulus, in MPa: 151.454, 151.454, (modified)

Moduli sequence, in MPa: 4210.6, 330.2, 175.8, 151.5,

Sensor	Distance, m	Deflection, mm	Hyperbola, mm	Error of Fit, %
1	0.000000	0.425000	0.000000	0.000000
2	0.300000	0.280000	0.358244	27.944310
3	0.600000	0.152000	0.141503	-6.905885
4	0.900000	0.085000	0.082183	-3.313615
5	1.200000	0.055000	0.055892	1.622527
6	1.500000	0.040000	0.041446	3.614994
7	1.800000	0.033000	0.032462	-1.631260

Overlay Design Values (for standard temperature, 21° C):

Milled-off recycling thickness, in mm:	40
Required overlay thickness for .35 mm of standard deflection:	
Minimum required thickness, in mm:	85.0
Moduli sequence, in MPa :	2436.6, 2820.7, 330.2, 175.8, 151.5
Thickness sequence, in mm:	85.0, 50.0, 45.0, 390.0
Penetration of the new asphalt:	100
Type of new asphalt concrete:	S-1
Total deflection, in mm:	0.34893
Subgrade deflection, mm:	0.15169
Subgrade stress, in kPa:	30.38
Subgrade micro-strain:	200.6

Example 2

Output File of *PROBE* Run
For up to three layers above subgrade (AC, base, subbase)

Load, in kN:	54.357
Radius, in mm:	150
Contact pressure, in kPa:	769
Layer thicknesses, in mm:	90 , 145 , 390 , 0 (subgrade)
Temperatures, at test (and adjusted), in ° Celsius:	16 (21)
Original penetration of asphalt:	88
Type of AC/modification factor for subgrade modulus :	S-1 / 1
(AC modulus, non-adjusted and adjusted, in MPa:	4210.6, 2820.7,)

Primary Response Parameters:	**Unadjusted**	**Adjusted**
Total deflection, in mm:	0.52600	0.41618,
Radius of AC curvature, in m:	186.4	235.5,
AC tensile micro-strain:	241.5	191.1,
Subgrade deflection, mm:	0.22599,	0.17108,
Subgrade stress, in kPa:	45.95,	34.78,
Subgrade micro-strain:	320.1	242.3,

η,ζ: 1.4830610, 0.0902239, J1 = 4
X(J1), Hem, in metres: 0.90000 0.80289

Subgrade modulus, in MPa: 143.518, 143.518, (modified)

Moduli sequence, in MPa: 4210.6, 341.5, 181.9, 143.5,

Sensor	Distance, m	Deflection, mm	Hyperbola, mm	Error of Fit, %
1	0.000000	0.526000	0.000000	0.000000
2	0.300000	0.352000	0.538001	52.841130
3	0.600000	0.189000	0.192458	1.829871
4	0.900000	0.107000	0.105483	-1.417637
5	1.200000	0.068000	0.068848	1.247008
6	1.500000	0.049000	0.049450	0.918695
7	1.800000	0.038000	0.037734	-0.699021

Overlay Design Values (for standard temperature, 21 ° C):

Milled-off recycling thickness, in mm:	40
Required overlay thickness for .35 mm of standard deflection:	
Minimum required thickness, in mm:	85.0
Moduli sequence, in MPa:	2436.6, 2820.7, 341.5, 181.9, 143.5
Thickness sequence, in mm:	85.0, 50.0, 45.0, 390.0
Penetration of the new asphalt:	100
Type of new asphalt concrete:	S-1
Total deflection, in mm:	0.34699
Subgrade deflection, mm:	0.15734
Subgrade stress, in kPa:	28.96
Subgrade micro-strain:	201.8

Appendix B/ Test Pit Example

One of the field trials of the PROBE program was at a C-SHRP test site (Highway 80, near Sarnia, Ontario), before rehabilitation by an overlay. The first of the FWD tests was carried out on top of the test pit before excavation. It is at this location that the dimensions of layer thicknesses are measured directly and are therefore more accurate and reliable than anywhere else along the test section. Indeed, the results from the four target load levels showed the least amount of scattering (less than half) compared to other test points. The AC layers were cracked and laminated, and the PROBE branch with the AC-layer modulus iteration was used. The Granular-A second-layer modulus was estimated and fixed at 345 MPa. The results of the FWD test and the PROBE analysis are presented in Table B1:

The stresses and deflections shown in Table B1 are adjusted for temperature (21°C) and load (40 kN). The last two rows show the maximum percent deviation from the calculated average.

Note also that the subgrade modulus decreases, slightly and consistently, with increasing load level, by a total of 7.2%. This is in concordance with commonly observed behaviour of all materials which are not linearly elastic.

The non-adjusted maximum deflection at the 8°C testing temperature for a 44.89 kN test load was 0.445 mm, to be reduced to 0.397 mm for 40 kN. The difference between this last value of 0.397 mm and the value of 0.4695 mm in Table B1 is due to the softening of the AC layer from a rise in temperature, from 8°C to 21°C, which corresponds to an estimated change in AC modulus from 852 MPa down to about 500 MPa. The other branch of PROBE overestimated the deflection by 10% (0.5161 mm) with a corresponding AC modulus change from 1100 to 476 MPa.

The accuracy of the subgrade moduli and of the adjusted subgrade parameters for the four calculations on the test pit data is greater than for the corresponding calculations of other points on the test site (except for the computed horizontal tensile strain in Layer 1). This improvement of consistency must be due to the greater accuracy in the measured layer thicknesses at the wall of the test pit excavated after the FWD test.

Normally, layer thicknesses are determined from a few core samples along the whole section under investigation, which is less reliable and less accurate. This affects mainly the adjusted parameters, whereas the effect on the subgrade modulus should be insignificant.

Table B1/ Test pit example, clay subgrade, granular bases

Test Load kN	Subgrade Modulus MPa	Total Deflect. * mm	Subgrade Deflect.* mm	Horiz. Strain* micro-	Subgrade* kPa	Stress/Strain* micro-
27.64	141.4	0.4691	0.1574	260.9	31.42	222.2
44.89	140.3	0.4695	0.1583	242.8	31.05	221.3
56.83	136.5	0.4712	0.1614	225.9	30.59	224.1
74.93	131.9	0.4733	0.1653	255.5	30.11	228.3
Average:	137.5	0.4708	0.1606	246.3	30.79	224.0
+%	+2.8 %	+0.53 %	+2.9 %	+5.9 %	+2.0 %	+1.8%
-%	-4.1 %	-0.36 %	-2.0 %	-8.3 %	-2.2 %	-1.2%

*adjusted values (for 40 kN and 21 °C)

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